

Paper Reference 9FM0/01
Pearson Edexcel
Level 3 GCE

Further Mathematics
Advanced
PAPER 1: Core Pure Mathematics 1

Thursday 25 May 2023 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green),
calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

X72794A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 8 questions in this Question Paper.
The total mark for this paper is 75.**

**The marks for EACH question are shown in brackets
– use this as a guide as to how much time to spend on
each question.**

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. The cubic equation

$$x^3 - 7x^2 - 12x + 6 = 0$$

has roots α , β and γ

Without solving the equation, determine a cubic equation whose roots are $(\alpha + 2)$, $(\beta + 2)$ and $(\gamma + 2)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p , q and r are integers to be found.

(Total for Question 1 is 5 marks)

2. (a) Write $x^2 + 4x - 5$ in the form $(x + p)^2 + q$ where p and q are integers.

(1 mark)

- (b) Hence use a standard integral from the formula book to find

$$\int \frac{1}{\sqrt{x^2 + 4x - 5}} dx$$

(2 marks)

- (c) Determine the mean value of the function

$$f(x) = \frac{1}{\sqrt{x^2 + 4x - 5}} \quad 3 \leq x \leq 13$$

giving your answer in the form $A \ln B$ where A and B are constants in simplest form.

(3 marks)

(Total for Question 2 is 6 marks)

3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$z_1 = -4 + 4i$$

- (a) Express z_1 in the form $r(\cos \theta + i \sin \theta)$,
where $r \in \mathbb{R}$, $r > 0$ and $0 \leq \theta < 2\pi$
(2 marks)

(continued on the next page)

3. continued.

$$z_2 = 3 \left(\cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12} \right)$$

(b) Determine in the form $a + ib$, where a and b are exact real numbers,

(i) $\frac{z_1}{z_2}$

(2 marks)

(ii) $(z_2)^4$

(2 marks)

(continued on the next page)

3. continued.

(c) Show on a single Argand diagram

(i) the complex numbers z_1 , z_2 and $\frac{z_1}{z_2}$

(ii) the region defined by

$$\{z \in \mathbb{C} : |z - z_1| < |z - z_2|\}$$

(4 marks)

(Total for Question 3 is 10 marks)

4. Prove by induction that for $n \in \mathbb{N}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & -2n \\ 0 & 1 \end{pmatrix}$$

(Total for Question 4 is 5 marks)

5. The line L_1 has equation

$$\frac{x+5}{1} = \frac{y+4}{-3} = \frac{z-3}{5}$$

The plane Π_1 has equation

$$2x + 3y - 2z = 6$$

- (a) Find the point of intersection of L_1 and Π_1
(2 marks)

The line L_2 is the reflection of the line L_1 in the plane Π_1

- (b) Show that a vector equation for the line L_2 is

$$\underline{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

where μ is a scalar parameter.

(5 marks)

(continued on the next page)

5. continued.

The plane Π_2 contains the line L_1 and the line L_2

- (c) Determine a vector equation for the line of intersection of Π_1 and Π_2
(2 marks)

The plane Π_3 has equation

$$\underline{r} \cdot \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = b \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the planes Π_1 , Π_2 and Π_3 form a sheaf,

- (d) determine the value of a and the value of b
(3 marks)

(Total for Question 5 is 12 marks)

6. Refer to the information for Question 6 in the Diagram Booklet.

Given that when $t = 0$ the volume of water in the tank is decreasing at a rate of 3 litres per minute, use the model to

- (a) show that the volume of water in the tank at time t satisfies

$$\frac{dV}{dt} = 3 - \frac{4}{1 + e^{0.8t}} - 0.4V$$

(3 marks)

- (b) Determine

$$\frac{d}{dt}(\arctan e^{0.4t})$$

(2 marks)

(continued on the next page)

6. continued.

Hence, by solving the differential equation from part (a),

(c) determine an equation for the volume of water in the tank at time t

Give your answer in simplest form as

$$\mathbf{V = f(t)}$$

(6 marks)

After 10 minutes, the volume of water in the tank was 8 litres.

(d) Evaluate the model in light of this information.

(1 mark)

(Total for Question 6 is 12 marks)

7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a) Explain why, for $n \in \mathbb{N}$

$$\sum_{r=1}^{2n} (-1)^r f(r) = \sum_{r=1}^n (f(2r) - f(2r-1))$$

for any function $f(r)$

(2 marks)

- (b) Use the standard summation formulae to show that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{2n} r \left((-1)^r + 2r \right)^2 = n(2n+1)(8n^2 + 4n + 5)$$

(6 marks)

(continued on the next page)

7. continued.

(c) Hence evaluate

$$\sum_{r=14}^{50} r \left((-1)^r + 2r \right)^2$$

(4 marks)

(Total for Question 7 is 12 marks)

8. Refer to the table for Question 8 in the Diagram Booklet.

A colony of small mammals is being studied.

In the study, the mammals are divided into 3 categories as shown in the Diagram Booklet.

- (a) State one limitation of the model regarding the division into these categories.

(1 mark)

A model for the population of the colony is given by the matrix equation

$$\begin{pmatrix} N_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} N_n \\ J_n \\ B_n \end{pmatrix}$$

where a and b are constants, and N_n , J_n and B_n are the respective numbers of the mammals in each category n months after the start of the study.

(continued on the next page)

8. continued.

At the start of the study the colony has breeders only, with no newborns or juveniles.

According to the model, after 2 months the number of newborns is 48 and the number of juveniles is 40

(b) (i) Determine the number of mammals in the colony at the start of the study.

(ii) Show that

$$a = 0.8$$

(4 marks)

(c) Determine, in terms of b ,

$$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}^{-1}$$

(3 marks)

(continued on the next page)

8. continued.

Given that the model predicts approximately **1015** mammals in total at the start of a particular month, and approximately **596** NEWBORNS, **464** JUVENILES and **437** BREEDERS at the start of the next month,

(d) determine the value of **b**, giving your answer to **2** decimal places.

(3 marks)

(continued on the next page)

8. continued.

It is decided to monitor the number of NEWBORN males and females as a part of the study.

Assuming that 42% of newborns are male,

(e) refine the matrix equation for the model to reflect this information, giving a reason for your answer.

(There is no need to estimate any unknown values for the refined model, but any known values should be made clear.)

(2 marks)

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
